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AXIAL HEAT DISTRIBUTION IN BROOKHAVEN REACTOR

By
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July 1951

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Associated Universities, Inc.

BROOKHAVEN NATIONAL LABORATORY
Upton, N. Y.

CONTENTS

DEFINITIONS AND STANDARD CONDITIONS

Symbols used in this report are tabulated below. Values marked with an asterisk (*) refer to the values of parameters under standard conditions. These quantities are altered in one of the graphs. Values marked with a plus sign (+) refer to the values of parameters altered for the purpose of comparison with experiment.

A	transverse cross-sectional area of graphite lattice available for heat flow = 378. cm^2
C_a	specific heat of air in channel = 0.24 cal/g-°C*
C_1	power transfer coefficient by conduction from metal to carbon = 0.0095 cal/cm-sec-°C*
C_2	power transfer coefficient by radiation from metal to carbon = 1.1×10^{-11} cal/cm-sec-°A^4*
D	4 x hydraulic radius of air channels = 2.27 cm
e	distance of flux maximum from gap face of pile = -3.5 cm*+
f	distance of plenum end of fuel rod from flux maximum = 339 cm*+
h	heat transfer coefficient from metal or carbon to air in channel = 0.0038 cal/cm^2-sec-°C*+
h'	heat transfer coefficient from carbon to air in gap = 0.00093 cal/cm^2-sec-°C*
K	cross-sectional area available for air flow in channel = 27.9 cm^2
k	thermal conductivity of graphite[1] = 0.17 cal/cm-sec-°C*+
k_a	thermal conductivity of air = 7.6×10^{-5} cal/cm-sec-°C
L	axial distance from flux maximum to point where flux extrapolates to zero = 381 cm*+
L_i	distance from center of pile to edge of orificing
L_i'	distance from center of pile to edge of loaded lattices, i^{th} direction
N	number of channels loaded = 1369+
P	power produced by reactor = 28 megawatts+
P_c	perimeter of the carbon = 21.3 cm
P_m	perimeter of the metallic fuel elements including the fins = 27.8 cm
Q	power generated in central half-channel = 6040 cal/sec+
Q'	power generated per unit length in central half-channel
R,S	coefficients of the two particular integrals of a certain linear, inhomogenous, first-order differential equation
s	transverse extrapolation distance (= 61 cm for comparison with experiment only)
T_a	temperature of air in channels
T_c	temperature of the carbon
T_m	temperature of the metallic fuel element
U	See equation (24) for its definition
V	See equation (22) for its definition
W	mass rate of air flow thru central channel = 138 g/sec*+
x	horizontal positional coordinate in a direction transverse to that of the fuel elements
y	vertical positional coordinate in a direction transverse to that of the fuel elements

[1]R.C. Garth and V.C. Sailor, "Thermal Conductivity of Graphite," BNL-69, November 28, 1949.

z axial positional coordinate in direction parallel to that of the fuel element

α fraction of all heat generated in the carbon = 0.05*+

μ viscosity of air = 225 micropoises

$\pi_{1,2}$ two roots of a quadratic equation. These roots are the coefficients of z in the exponentials corresponding to the solutions of a certain linear, homogenous, first-order differential equation

ω $\pi/2L = 4.12 \times 10^{-3}/\mathrm{cm}$

PREFACE

The axial temperature distribution is calculated in a pile with a central air gap. The effect of various geometrical and thermal constants on this distribution is presented in Figures 4 through 10, and a tabular summary is presented in Figure 11. The coefficient of heat transfer to the air and the specific heat of the air are the most important parameters. The fraction of heat generated in, and the heat conducted or radiated to, the graphite and the consideration of the actual flux distribution are of secondary importance. The thermal conductivity of graphite and the heat transferred to the gap are of negligible importance.

A simple theory is compared with a more elaborate theory in Figure 3. The air temperature in all cases can be computed just as well from the simple theory as from the more elaborate one. The temperatures of the fuel elements, the air and the graphite at the plenum end are all very nearly equal, and the elaborate and simple theories lead to essentially the same result for this temperature. For the Brookhaven reactor at 28-Mw power, the elaborate theory predicts the fuel element temperature at the gap to be some 20°C lower and the graphite temperature to be some 40°C higher than does the simple theory. These differences gradually vanish as one moves along toward the plenum chamber.

Finally, in Figure 12, an attempt is made to compare theory and experiment. The agreement is poor because of experimental difficulties and a theoretical difficulty as explained in the final section, Numerical Results and Conclusions.

AXIAL HEAT DISTRIBUTION IN BROOKHAVEN REACTOR

EFFECT OF VARIOUS GEOMETRICAL AND THERMAL CONSTANTS AND COMPARISON WITH EXPERIMENT

Introduction and Approximations

The primary purpose of this report is to calculate the temperature distribution along the air cooling channels in the Brookhaven reactor. Not only is the temperature distribution needed in order to design a reactor, but it is requisite for the evaluation of the temperature coefficient, xenon poisoning, stability under flashes, etc.

The secondary purpose of this study is to determine the influence on this distribution of various geometrical and thermal constants so that simplifications can be made in future calculations by neglecting unimportant considerations.

A third purpose is to compare a simple theory and a more elaborate theory. Fourthly, the elaborate theory is to be compared with experiment. Unfortunately, a valid comparison is almost impossible because of experimental reasons and because the theory in at least one respect is inadequate. The reasons are presented in the section, Numerical Results and Conclusions.

The Brookhaven reactor consists of a graphite cube split down a central plane. Air flows into this split or gap and out of the air channels, as shown in Figure 1. The air channels of the Brookhaven reactor have their flow restricted or orificed in such a a way that the mass rate of flow through any channel is proportional to the heat energy produced in that channel. This fact introduces the simplification that the reactor is,[2] on the average, approximately isothermal in any plane parallel to the gap. We may, therefore, restrict ourselves to a consideration of the central channel and its associated lattice. Were it not for the orificing, this channel would be the hottest. We see, also, that heat flow in directions parallel to the gap may be neglected in a first approximation. We consider, therefore, only the flow of heat axially along the channels and the interchange of heat into and out of the ends of the graphite lattice.

The problem of the temperature distribution within the fuel element itself has been considered elsewhere and will not be treated here.[3] The temperature of the fuel element will be characterized by its surface temperature at any axial point.

The radiation of heat from the fuel elements is proportional to the fourth power of the temperature. Because this nonlinear dependence introduces considerable mathematical complications, we linearize this dependence by considering just the first two terms in a Taylor series expansion. It is to be noted that only about 20% of all the heat is transferred by radiation (to the graphite), most of the remainder being

[2] It is to be noted that the heat transfer coefficient is proportional to the mass rate of air flow to the 0.8 power. Hence, the periphery is slightly cooler.

[3] M. Clark, Jr., "Temperature of and Stresses in Cylindrical Fuel Elements During Pile Flashes," BNL-86.

transferred directly to the air. The linearization approximation will not introduce a serious error.

Heat may be transferred to the graphite from the fuel elements by radiation and by conduction, and from the air by conduction. Further, heat may be generated in the graphite by the slowing down of neutrons and by the absorption of gamma rays.

The air in the channels is so highly turbulent that it may be considered thoroughly mixed across any cross section transverse to the channel. The air may at any axial point be characterized by a single temperature.

We shall assume the power generated varies cosinusoidally from the center of the gap.[4]

Simple Method of Calculation

Before formulating the more accurate method of calculation, let us consider a simple first approximation. We neglect all forms of heat transfer except the conduction from the fuel elements to the air stream and from the carbon to the air stream. Three statements serve to formulate the equations:

1. The power generated by the metal is equal to that carried away by the air:

$$(1-\alpha)\frac{\pi Q}{2L}\cos\frac{\pi z}{2L} = hP_m\,(T_m - T_a), \tag{1}$$

where

α = fraction of all energy of fission released in carbon
Q = power generated in central half-channel at flux maximum
L = half-length of extrapolated pile along fuel element direction
z = distance of point of interest along a fuel element measured from the center of the reactor
h = power transfer coefficient to the air stream from either metal or carbon
P_m = perimeter of metal fuel element including fins
T_m = temperature of the metal
T_a = temperature of the air

2. The power generated in the carbon is equal to the power carried away by air:

$$\alpha\frac{\pi Q}{2L}\cos\frac{\pi z}{2L} = hP_c\,(T_c - T_a), \tag{2}$$

where

[4] J. Chernick, I. Kaplan, J. Kunstadter, V. Sailor and C. Williams, "An Experimental and Theoretical Study of the Subcritical BNL Reactor," BNL-60.

P_c = perimeter of carbon
T_c = temperature of carbon

3. The power absorbed by the air equals the power absorbed from the carbon and from the metal:

$$C_a W \frac{dT_a}{dz} = hP_c\ (T_c - T_a) + hP_m\ (T_m - T_a), \tag{3}$$

where

C_a = specific heat of the air
W = mass flow rate of air in the central channel

The above equations are readily solved by finding one temperature in terms of the others and subsequently eliminating that temperature in the remaining equations. The results are:

$$T_a = \frac{Q}{C_a W} \sin \frac{\pi z}{2L} \tag{4}$$

$$T_m = \frac{(1-\alpha)\pi Q}{2hP_m L} \cos \frac{\pi z}{2L} + T_a \tag{5}$$

$$T_c = \frac{\alpha \pi Q}{2hP_c L} \cos \frac{\pi z}{2L} + T_a \tag{6}$$

The zero of the temperature scale is taken to be the inlet temperature of the air.

The mass flow rate of air in the central channel is easy to determine. Because of the orificing, the air flow in a channel varies as $\cos(\pi x_i/2L_1)\cos(\pi y_i/2L_2)$, where the channel is located at (x_i, y_i). Hence, the total flow of air is:

$$2W \sum_{i,j} \cos\left(\frac{\pi x_i}{2L_1}\right) \cos\left(\frac{\pi y_i}{2L_2}\right). \tag{7}$$

The summation may be replaced by an integration; the total air flow is:

$$\frac{2WN}{4L_1L_2} \int_{-L_1}^{L_1} dx \int_{-L_2}^{L_2} dy \cos\left(\frac{\pi x}{2L_1}\right) \cos\left(\frac{\pi y}{2L_2}\right) \tag{8}$$

$$= \frac{8NW}{\pi^2}, \tag{9}$$

where N = number of channels loaded into a rectangular array, $2L_1$ by $2L_2$ in size. Since the total mass flow rate of air is known, it is simple to determine W from equation (9).

The power transfer coefficient is determined[5] from the relation:

$$\frac{hD}{k_a} = 0.023 \left(\frac{DW}{\mu K}\right)^{0.8} \left(\frac{C_a \mu}{k_a}\right)^{0.4}, \tag{10}$$

where

k_a = thermal conductivity of air
D = 4 x hydraulic radius
μ = viscosity of air
K = cross-sectional area available for air flow
C_a = specific heat of air at constant pressure

Finally, Q is determined from the total power P:

$$P = 2 \sum_{i,j} \int_0^L dz \frac{\pi Q}{2L} \cos \frac{\pi z}{2L} \cos \frac{\pi x_i}{2L_1} \cos \frac{\pi y_i}{2L_2}$$

$$= \frac{8QN}{\pi^2} \tag{11}$$

(cf. equation (9)).

Elaborate Method of Calculating Temperature Distribution

We turn now to a more elaborate method of calculation. Three statements of energy conservation suffice to establish the equations of heat flow:

1. The power made in the fuel element equals the power carried away by conduction to the carbon plus that carried away by radiation to the carbon plus that carried away by convection to the air:

$$(1-\alpha)\ Q' \cos \frac{\pi z}{2L} = C_1\ (T_m - T_c) + C_2\ (T_m^4 - T_c^4) + hP_m\ (T_m - T_a), \tag{12}$$

where

Q' = power generated at flux maximum per unit length of fuel element $= \pi Q/2L$
z = axial distance to point of interest from maximum of neutron flux

[5]W.H. McAdams, Heat Transmission. 2nd ed., McGraw-Hill Book Co., Inc., 1942, p. 168.

C_1 = power conduction coefficient from metal to carbon
C_2 = power radiation coefficient from metal to carbon

2. The power produced in an axial section of the carbon equals the change in power transported parallel to a channel plus the power conducted to the metal plus the power lost by radiation to the metal plus the power carried away by the air:

$$\alpha Q' \cos\frac{\pi z}{2L} = - kA\frac{d^2T_c}{dz^2} - C_1 (T_m - T_c) - C_2 (T_m^4 - T_c^4) + hP_c (T_c - T_a) , \tag{13}$$

where

k = conductivity of carbon
A = transverse cross-sectional area of carbon in one lattice

3. The increase in temperature of the air multiplied by the heat capacity and the mass rate of air flow equals the power given to the air from the metal plus the power given to it from the carbon:

$$C_a W \frac{dT_a}{dz} = hP_m (T_m - T_a) + hP_c (T_c - T_a) . \tag{14}$$

The boundary conditions are:

1. The power flowing axially from the graphite to the air in the gap is proportional to the temperature difference between the carbon and the air at that point. At $z = -|e|$

$$k\frac{dT_c}{dz} = h' (T_c - T_a) , \tag{15}$$

where

$|e|$ = the distance from the maximum of the neutron flux to the gap face (see Figure 2);
h' = the heat transfer coefficient to the air in the gap from the graphite

2. The power flowing axially from the graphite to the air in the plenum chamber is proportional to the temperature difference between the carbon and the air at that point. However, the temperature difference is small and the power transfer coefficient is small, because of the low transverse air velocity. Hence, we may neglect the power transfer to the plenum from the face of the graphite. A further approximation is in order. The fuel rods end at a point short of the pile face, and the flux extrapolates to zero at a distance intermediate between the ends of the rods and the plenum chamber face of the pile. Since much of the heat produced in the carbon is produced by heat radiation from the fuel elements, by slowing down neutrons, and by stopping gamma rays, we take the point where the flux extrapolates to zero as the point to apply the zero power transfer condition. At $z = L$,

$$k \frac{dT_c}{dz} = 0. \tag{16}$$

Any other treatment would require a complicated analysis in several dimensions and would make essentially no difference.

The linearized equations become:

$$(1-\alpha)\, Q' \cos\left(\frac{\pi z}{2L}\right) = C_1\, (T_m - T_c) + 4C_2\, \overline{T_m^3}\, (T_m - T_c) + hP_m\, (T_m - T_a) \tag{17}$$

$$\alpha Q' \cos\left(\frac{\pi z}{2L}\right) = -kA \frac{d^2 T_c}{dz^2} - C_1\, (T_m - T_c) - 4C_2\, \overline{T_m^3}\, (T_m - T_c) + hP_c\, (T_c - T_a) \tag{18}$$

$$C_a W \frac{dT_a}{dz} = hP_m\, (T_m - T_a) + hP_c\, (T_c - T_a) \tag{19}$$

These equations are solved by the tedious process of systematically solving for one temperature in terms of the others and of eliminating that temperature from all other equations. We may arrive at an inhomogeneous, third order, linear differential equation for the air temperature. This equation is solved by the usual methods. The complete solution consists of a solution of the homogeneous equation which involves a positive and negative exponential, plus a solution of the inhomogeneous equation consisting of a linear combination of sin ωz and cos ωz ($\omega = \pi/2L$). The two roots of the homogeneous equation are:

$$\pi_{1,2} = \frac{-kA \pm \sqrt{(kA)^2 + 4C_a WV}}{2V}, \tag{20, 21}$$

where

$$V = \frac{kAC_a W\, (C_1 + 4C_2\, \overline{T_m^3} + hP_m)}{hP_m\, (C_1 + 4C_2\, \overline{T_m^3}) + hP_c\, (C_1 + 4C_2 \overline{T_m^3} + hP_m)}. \tag{22}$$

The two roots π_1 and π_2 provide the coefficient of z in the two exponentials.

The coefficients of the two terms in sin ωz and cos ωz, corresponding to the inhomogenous solution are determined by substitiution of the solution into the differential equation. The coefficient S of the cosine term is:

$$S = \frac{U\, (V\omega^2 + C_a W)}{(V\omega^2 + C_a W)^2 + (kA)^2\, \omega^2}, \tag{23}$$

where

$$U = Q' \left[1 + \frac{kA\omega^2 hP_m \ (1-\alpha)}{hP_m \ (C_1+4C_2\overline{T_m^3}) \ + \ hP_c \ (C_1+4C_2\overline{T_m^3}+hP_m)} \right]. \tag{24}$$

The coefficient, R, of the sine term is:

$$R = \frac{-kA\omega U}{(V\omega^2+C_aW)^2 + (kA\omega)^2}. \tag{25}$$

The two boundary conditions (15) and (16) serve to determine the coefficients a/π_1 and b/π_2 of the two exponential terms, each of which is a solution of the homogeneous differential equation for the air temperature. The following two equations may be solved simultaneously for a and b:

$$k\left[e^{-\pi_1|e|}a + e^{-\pi_2|e|}b\right] + \frac{C_aW\left[hP_m+C_1+4C_2\overline{T_m^3}\right]\left[(\pi_1k-h')e^{-\pi_1|e|}a + (\pi_2k-h')e^{-\pi_2|e|}b\right]}{h\left[hP_mP_c + (C_1+4C_2\overline{T_m^3})(P_c+P_m)\right]} =$$

$$k\left[R \sin \omega|e| - S \cos \omega|e|\right] + \frac{C_aW\left[hP_m + C_1 + 4C_2\overline{T_m^3}\right]\left[(h'S-k\omega R) \cos \omega|e| + \right.}{h\left[hP_mP_c + (C_1+4C_2\overline{T_m^3})(P_c+P_m)\right]}$$

$$\frac{\left.-(h'R+k\omega S) \sin \omega|e|\right] + [1-\alpha]\, hP_mQ'\left[-h' \cos \omega|e| + \omega k \sin \omega|e|\right]}{} \tag{26}$$

$$\left[e^{\pi_1L}a + e^{\pi_2L}b\right] + \frac{C_aW\left[hP_m + C_1 + 4C_2\overline{T_m^3}\right]\left[\pi_1e^{\pi_1L}a + \pi_2e^{\pi_2L}b\right]}{h\left[hP_mP_c + (C_1+4C_2\overline{T_m^3})(P_c+P_m)\right]}$$

$$= -\left[S \cos \omega L + R \sin \omega L\right] - \frac{\omega C_aW\left[hP_m + C_1 + 4C_2\overline{T_m^3}\right]\left[R \cos \omega L - S \sin \omega L\right] +}{h\left[hP_mP_c + (C_1+4C_2\overline{T_m^3})(P_c+P_m)\right]}$$

$$\frac{+ [1-\alpha]\, hP_mQ'\omega \sin \omega L}{}. \tag{27}$$

Q' is determined from the total power:

$$P = 2Q' \int_e^f dz \cos \frac{\pi z}{2L} \sum_{i,j} \cos\left(\frac{\pi x_i}{L'+s}\right) \cos\left(\frac{\pi y_i}{L'+s}\right), \tag{28}$$

where

f = distance from flux maximum to end of fuel element
L′ = distance from center of pile to edge of loaded lattices
s = transverse extrapolation distance.

It is found that

$$Q' = \frac{\pi^3 P}{16\ LN\ [1+s/L']^2 \left[\sin \pi/2\ (1+s/L')\right]^2 [\sin(\pi f/2L) + \sin(\pi|e|/2L)]} . \tag{29}$$

The power transfer coefficients, one for the air in the gap, one for the air in the channels, are found from equation (10). The constants C_1 and C_2 are found from experiments, and $\overline{T_m^3}$ may be determined from the simple calculation discussed earlier. The mass rate of air flow is found from equation (9).

The solutions are as follows:

$$T_a = \frac{a}{\pi_1}(e^{\pi_1 z}-1) + \frac{b}{\pi_2}(e^{\pi_2 z}-1) + \frac{S}{\omega}\sin \omega z + \frac{R}{\omega}(1-\cos \omega z) \tag{30}$$

$$T_c = T_a + \frac{C_a W\left[hP_m + C_1 + 4C_2\overline{T_m^3}\right]\left[e^{\pi_1 z} a + e^{\pi_2 z} b + S\cos\omega z + R\sin\omega z\right] + }{h\left[hP_m P_c + (C_1+4C_2\overline{T_m^3})(P_c+P_m)\right]}$$

$$\frac{-\ Q' hP_m\ (1-\alpha)\cos\omega z}{\qquad} . \tag{31}$$

$$T_m = T_a + \frac{C_a W\left[C_1 + 4C_2\overline{T_m^3}\right]\left[e^{\pi_1 z} a + e^{\pi_2 z} b + S\cos\omega z + R\sin\omega z\right] + }{h\left[h^2 P_m P_c + (C_1+4C_2\overline{T_m^3})(P_c+P_m)\right]}$$

$$\frac{+\ Q' hP_c\ (1-\alpha)\cos\omega z}{\qquad} \tag{32}$$

Numerical Results and Conclusions

The purpose of this report is to study the effects of various parameters on the heat distribution in the Brookhaven reactor. The results of this study are summarized in tabular form in Figure 11, and in graphical form in Figures 3-10, and 12.

Each of the graphical Figures 3-10 consists of six graphs. Three of these graphs (solid lines) relate to the quantity being studied and the remaining three (dashed lines) are presented for comparison. These latter three refer to standard values of the parameters and are based on the simple method of calculation. In each set of three graphs, one describes the fuel element temperature, a second illustrates the graphite moderator temperature, and the third relates to the air temperature. The air temperature as it enters a channel from the gap is taken to be zero in all cases. The values of the parameters taken as standard will be found in the table defining our symbols. For those parameters altered in each graphical figure, both the standard value and the altered value are quoted. Unstated values are standard.

The tabular Figure 11 is a comparison of the fuel element, moderator, and air temperatures when one parameter at a time is changed to their temperatures under standard conditions as computed from the more elaborate theory. The standard and altered values of the parameter changed are indicated in each case. The temperatures of the fuel elements are compared at three points: at the gap, at the plenum end of the pile, and at the maximum (which may be shifted under the altered conditions). The graphite temperatures and the air temperatures are compared at the gap, at the middle of the reactor, and at the plenum end. A positive sign means the temperature under altered conditions is higher than under standard conditions.

The results allow a few generalizations:

1. The air temperature can be computed as well from the simple theory as from the elaborate theory.

2. The metal, graphite, and air are equally hot at the plenum end of a channel.

3. The graphite and air have no maximum temperature, at least of any consequence.

4. For about 30% of a channel length from the plenum end, the graphite temperature is essentially constant.

5. The maximum metal temperature occurs about half-way along a channel.

6. The heat transfer coefficient to the air from the graphite or metal and the specific heat of the air are the two parameters of major importance.

7. The flux distribution, the fraction of heat generated in the carbon and the heat conducted or radiated to the graphite are of secondary importance.

8. The thermal conductivity of graphite and the heat transfer coefficient to the gap are relatively unimportant, except for the graphite temperature at the gap face.

Finally, Figure 12 is a comparison of theory and experiment as regards fuel element temperature for two channels. The agreement is poor, even though the parameters are as representative as possible. Several reasons for the poor agreement may be adduced:

1. We assume the heat transfer coefficient to the air in a channel to be constant. Actually, the turbulence is greater near the gap, so the heat transfer coefficient is

larger, and the metal is cooler than our calculations indicate. This explanation is one of the few which will account for the rapid increase in fuel element temperature near the gap.

2. Errors occur in the measurement of temperatures. Thermocouples were mounted on the aluminum fins of the cartridges and were located directly in the air stream. These thermocouples cause local perturbations in the air flow, hence in the heat transfer coefficient and in the metal temperature.

3. Errors take place in the measurement of power.

4. We assumed all air going through the reactor goes through the channels; 10% may leak around. The total air flow was measured. The pile was orificed for 692 channels in a double cosine manner. Of these, 112 of the peripheral channels were plugged and assumed to transmit no air. Of the remaining 580 channels, 2.5 channels had no fuel element in them and no orifice. We assumed advisedly that these transmitted twice as much air as an unorificed, unplugged channel loaded with a fuel cartridge. Not only errors in the air flow per channel occur in the foregoing considerations, but air may leak around the pile at points other than the channels. Evidence for this leakage is provided by the unexpectedly low graphite temperature. Any leakage would affect the two important parameters of air flow per channel and the heat transfer coefficient. A 10% decrease in air flow would affect the maximum metal temperature by roughly 20°C.

In the calculations relating to the comparison with experiment, we assumed the reflector savings to be 61 cm on each side of the reactor.[4] The maximum of the flux was calculated[6] to be 80 cm from the gap center.

In conclusion, we would like to point out a few simplifications:

1. If the numerical coefficients are examined, especially the exponential factors in equation (26), to a very high degree of approximation we may put the coefficient of a equal to zero and solve directly for b.

2. Likewise, in equation (27) we may take the coefficient of b equal to zero and solve directly for a.

3. In view of the unimportance of the heat transfer coefficient h' to the gap air, h' may be set equal to 0, unless the finer details of the graphite temperature are desired right at the gap.

4. Because the thermal conductivity k of the graphite is of negligible importance, equation (26) becomes finally

[6] J. Chernick, "The Critical Experiment," #6 of serial reports on start-up experiments. BNL Log No. C-4736, April 2, 1951.

$$
\begin{aligned}
b = e^{\pi_2 |e|} & \left\{ 1 + \frac{C_a W \pi_2 \left[hP_m + C_1 + 4C_2 \overline{T_m^3} \right]}{\left[h^2 P_m P_c + (C_1 + 4C_2 \overline{T_m^3})(hP_c + hP_m) \right]} \right\}^{-1} * \\
* & \left\{ R \sin \omega|e| - S \cos \omega|e| - \frac{C_a W \omega \left[hP_m + C_1 + 4C_2 \overline{T_m^3} \right] \left[R \cos \omega|e| + S \sin \omega|e| \right] + (1-\alpha)\, \omega Q' hP_m \sin \omega|e|}{\left[h^2 P_m P_c + (C_1 + 4C_2 \overline{T_m^3})(hP_c + hP_m) \right]} \right\}.
\end{aligned}
\tag{33}
$$

5. Likewise, equation (27) becomes

$$
\begin{aligned}
a = e^{-\pi_1 L} & \left\{ 1 + \frac{C_a W \pi_1 \left[hP_m + C_1 + 4C_2 \overline{T_m^3} \right]}{\left[h^2 P_m P_c + (C_1 + 4C_2 \overline{T_m^3})(hP_c + hP_m) \right]} \right\}^{-1} * \\
* & \left\{ -R \sin \omega L - S \cos \omega L - \frac{C_a \omega W \left[hP_m + C_1 + 4C_2 \overline{T_m^3} \right] \left[R \cos \omega L - S \sin \omega L \right] + (1-\alpha)\, \omega Q' hP_m \sin \omega L}{\left[h^2 P_m P_c + (C_1 + 4C_2 \overline{T_m^3})(hP_c + hP_m) \right]} \right\}.
\end{aligned}
\tag{34}
$$

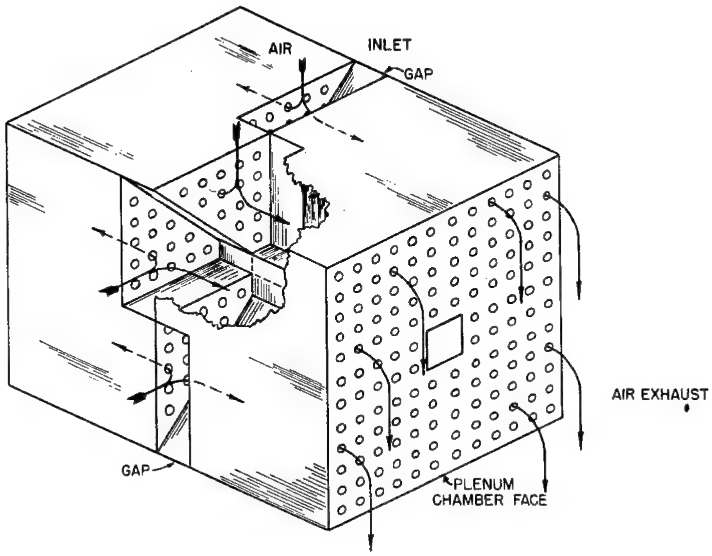
6. The air temperature T_a is found from equation (30). We note the first term is of consequence only at the plenum end of the channel; the second term is important only at the gap end of a channel. The sine term is by far the most significant, being at all times one and a half orders of magnitude more than all others combined.

7. The carbon temperature, T_c, is found from equation (31). The term in a is important only at the plenum end of a channel; that in b is important only at the gap end; the cosine terms are considerably more important than the others, usually by as much as one order of magnitude. The term T_a and the remaining expression for T_c are of comparable size.

8. The metal temperature, T_m, is found from equation (32). As before, the a term is important only at the plenum end, the b term at the gap end, and the cosine term dominates all others usually by at least an order of magnitude. The term in T_a and the remaining expression for T_m are of comparable magnitude.

Acknowledgments

The author is pleased to thank Dr. Irving Kaplan and Mr. Jack Chernick for several valuable discussions. Mrs. Jean Dominy is to be thanked for some of the numerical work.



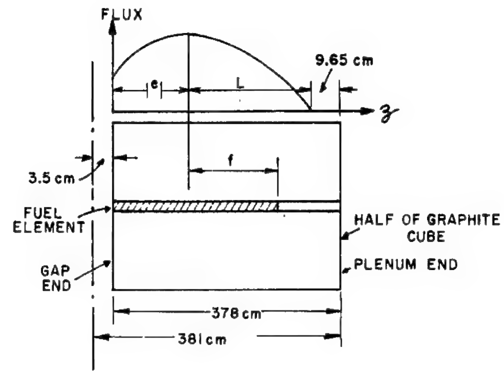


Figure 1 (Left). Air flow pattern in BNL reactor. BNL Log No. D-89.
Figure 2 (Above). Various dimensions of half-reactor. BNL Log No. D-1837.

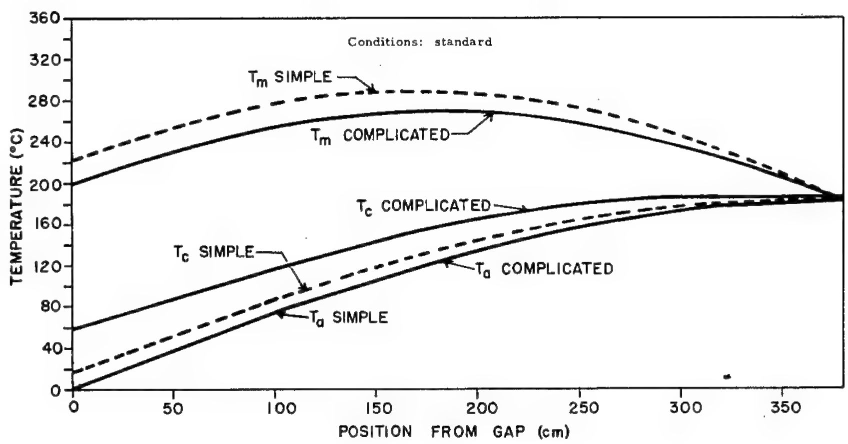


Figure 3. Comparison of simple theory and complicated theory. BNL Log No. D-1834.

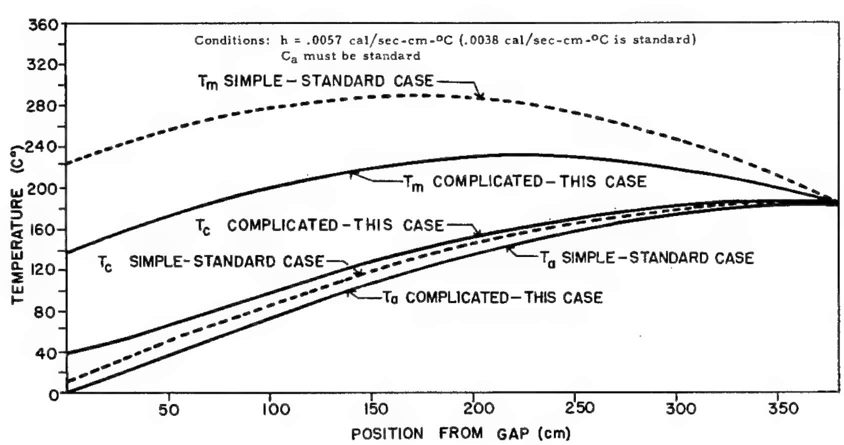


Figure 4. Effect of heat transfer coefficient to the air in channel. BNL Log No. D-1835.

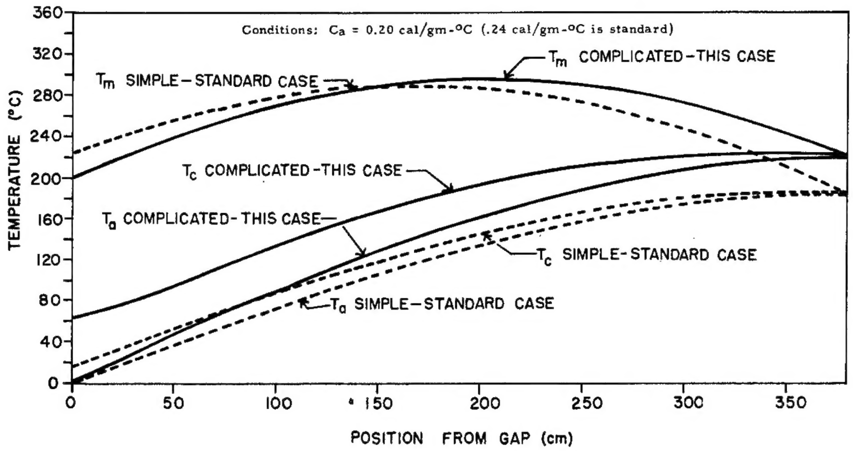


Figure 5. Effect of specific heat of air. BNL Log No. D-1836.

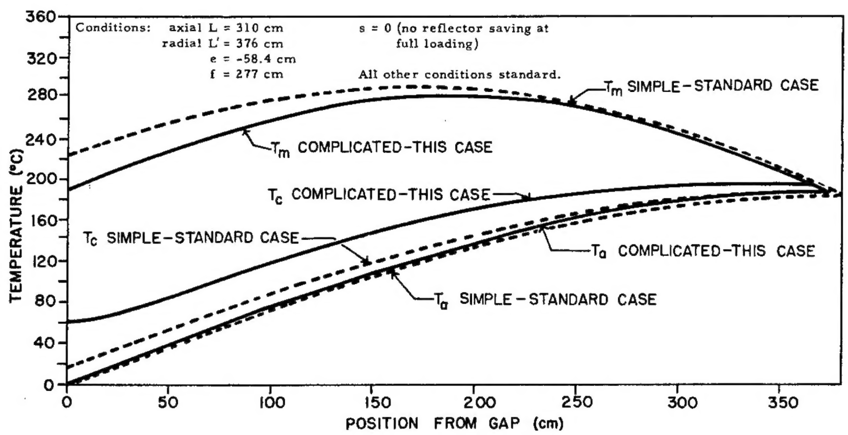


Figure 6. Effect of actual flux distribution. BNL Log No. D-1838.

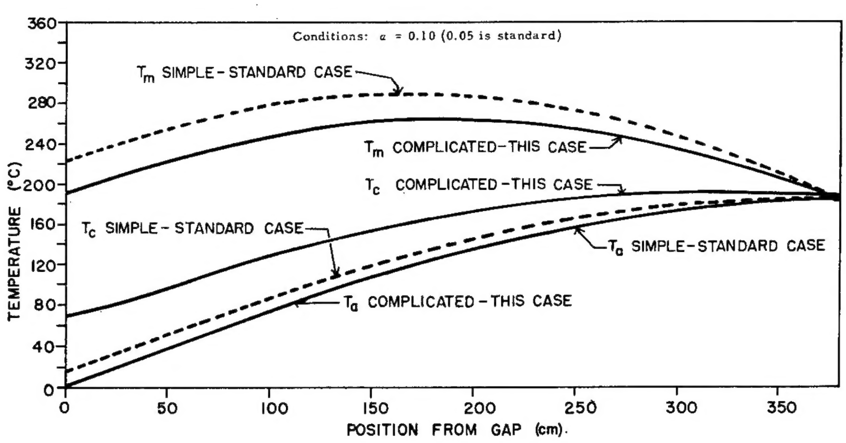


Figure 7. Effect of heat generated in carbon. BNL Log No. D-1839.

GPO 822898 - 2

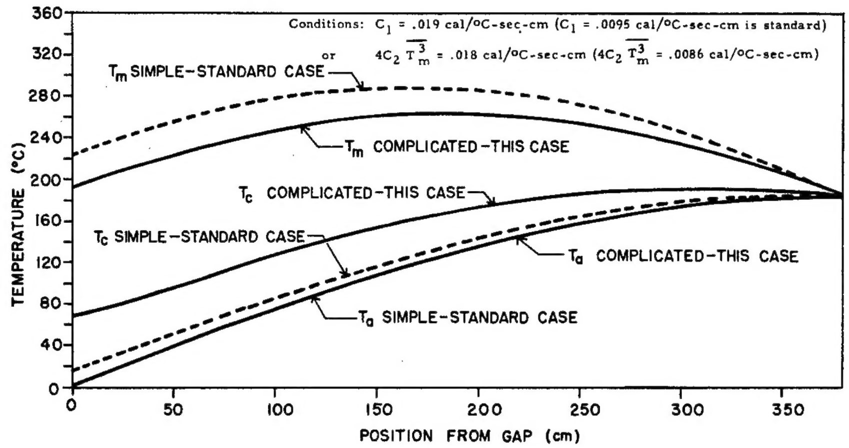


Figure 8. Effect of heat radiated or conducted to carbon. BNL Log No. D-1840.

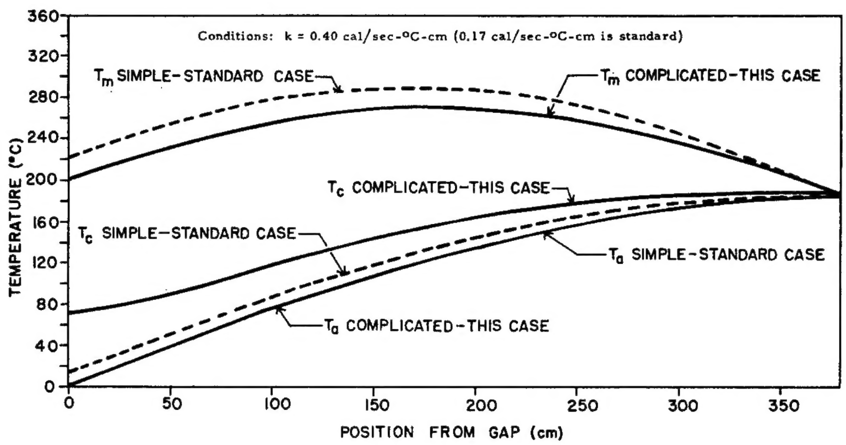


Figure 9. Effect of heat conductivity of carbon. BNL Log No. D-1841.

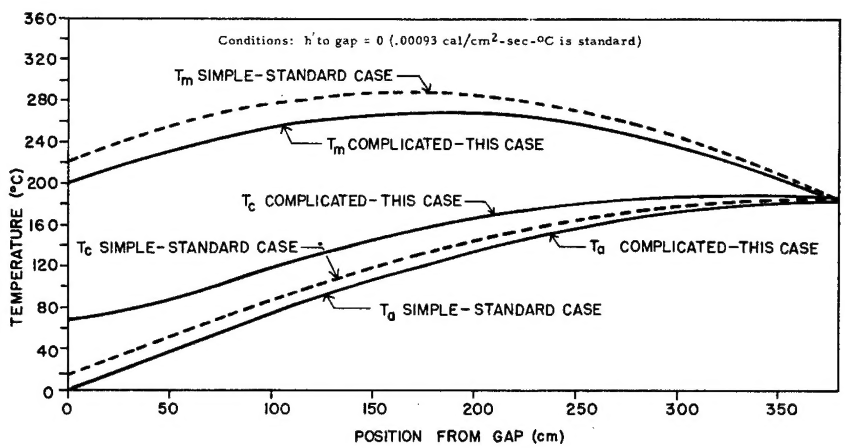


Figure 10. Effect of no heat being transferred to gap air. BNL Log No. D-1842.

Summary of Temperatures Under Various Conditions in Reactor*

Fig.	Entity Studied	Altered Value	Standard Value	Change in Temperatures (°C) (Altered Value-Standard Value)									Remarks
				Fuel Element			Carbon			Air			
				Gap	Max.	Plenum	Gap	Middle	Plenum	Gap	Middle	Plenum	
3	Theory	Elaborate	Simple	-20	-19	0	33	21	5	0	0	0	T_m = Max. shifted 8% of channel length towards plenum by elaborate theory T_c = Plateau at plenum, no max. T_a = Same in two theories
4	Heat transfer coefficient to channel air	0.0057 cal/sec-cm^2-°C. C_a must be standard	0.0038 cal/sec-cm^2-°C	-62	-42	0	-21	-14	-2	0	0	0	T_m = Max. at 60% of channel length T_c = Plateau, no max.
5	Specific heat of air	0.20 cal/g-°C	0.24 cal/g-°C	0	26	36	3.5	27	35	0	26	35	T_m = Max. at 50% of channel length T_c = Plateau, no max.
6	Location flux max. (Actual distribution)	58 cm along channel (correct)	0.0 cm (Max. shift neglected)	-10	11	5	0	4	5	0	2	5	T_m = Max. at 50% of channel length T_c = Plateau, no max. low farther along channel from gap No reflector savings at full loading assumed
7	Fraction of heat generated in carbon	10%	5%	-9	-5	0	9	8	2	0	0	0	T_m = Max. at 50% of channel length T_c = Very slight max. near plenum
8	Heat conducted to C or radiated to C	0.019 cal/sec-cm-°C 0.018 cal/sec-cm-°C	0.0095 cal/sec-cm-°C 0.0086 cal/sec-cm-°C	-9	-6	-1	9	8	2	0	0	0	T_m = Max. at 50% of channel length T_c = Very slight max. near plenum
9	Conductivity of C	0.40 cal/sec-cm-°C	0.17 cal/sec-cm-°C	0	0	0	12	0	0	0	0	0	T_m = Max. at 50% of channel length T_c = Hotter for about 30 cm
10	Heat transfer coefficient to gap air	0.00 cal/sec-cm^2-°C	0.00093 cal/sec-cm^2-°C	0	0	0	9	1	0	0	0	0	T_m = Max. at 50% of channel length

*Effects are arranged for the most part in order of importance. (Parameters are changed by an amount having some significance; usually extremes of reasonable values are taken.)

Figure 11.

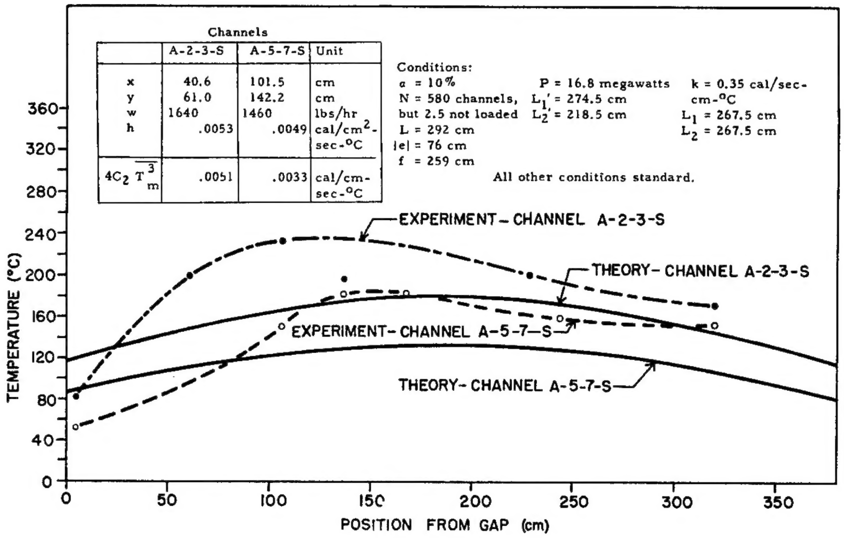


Figure 12. Comparison of experimental metal temperatures with complicated theory. BNL Log No. D-1844.